Balance of Payments Constrained Non-Scale Growth and the Population Puzzle

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Abstract

New growth theories suffer from their counterfactual prediction that a higher (rate of) population (growth) generally implies a higher growth rate of per capita income. This *population puzzle* is solved by introducing imported intermediates and an exogenously growing world export demand creating a Keynesian balance of payments constraint in a neoclassical model of non-scale growth. It is shown among other things that the growth rate of per capita consumption may decline or increase in the rate of population growth depending on the relative magnitudes of particular elasticities. The growth rate of per capita income measured in terms of the intermediate produced abroad will fall in the population growth rate if world export demand is price-inelastic. This elasticity is also decisive in whether the implications of the model are more *Keynesian* or more *neoclassical*.

1 Introduction

The new growth theory succeeded in solving an important problem of the basic neoclassical growth model, namely its failure to endogenously explain long-run growth of per capita income. As Jones (1995) has pointed out, however, Romer's (1990) seminal R&D model and many of its followers involve the problem of *scale effects*, according to which an increase in the size or scale of the economy permanently increases its long-run growth rate. The size of an economy is typically measured by its population. If population grows at a constant rate, the scale effect will imply that the growth rate of national income per capita itself grows at the rate of population growth. This prediction is clearly at odds with empirical evidence. Jones (1995) has shown that a plausible model can be constructed which maintains some of the features of the R&D models but does not involve these scale effects. Such a model, however, does also alter some of the main implications of the new growth theory. In particular, the long-run growth rate turns out to depend on parameters usually taken to be invariant to government policy. Nevertheless, such *non-scale* growth models endogenously explain technical progress and are therefore also referred to as *semi-endogenous* growth models.[1] As a general feature of non-scale growth models, long-run per capita growth rates are proportional to the growth rate of population. Hence, while the scale effect of the *level* of population is eliminated, it generally shows up with respect to the *growth rate* of population. A higher rate of population growth implies a higher steady state growth rate of per capita income.[2]

The general steady state properties of non-scale growth models, which are not restricted to R&D based approaches but do also include investment based models, are analyzed by Eicher and Turnovsky (1999). It is straightforward to show that these properties are, with one exception, reasonably consistent with the so-called *stylized facts* about economic growth as formulated by Kaldor (1961) and extended by Romer (1989). The exception pertains to the negative correlation between population growth rates and the growth of per capita income. Although it may be too much to refer to this assertion as a *stylized fact*, a number of cross-country studies have shown that population growth and growth of per capita output are either uncorrelated or even negatively correlated (cf. e.g. De Long and Summers, 1991; Mankiw et al., 1992). According to Goodfriend and McDermott (1995, p. 128), the empirical relationship between population and per capita product appears weak,

[1] It should be noted that a model of non-scale growth (without scale-effects) need not be a model of semi-endogenous growth (without political influence on the growth rate), nor the other way around. In the absence of knife-edge conditions with respect to particular parameters, however, both properties usually go hand in hand.

[2] In order to eliminate the prediction that long-run per capita growth rates are zero in the absence of population growth and independent of policy instruments, a second class of non-scale growth models has been invented (cf. e.g. Peretto, 1998, who is explicitly concerned with population growth). Jones (1999) provides a convenient comparison of both types of models. As it stands, the second class of non-scale models shares the feature that a higher rate of population growth implies a higher rate of per capita income growth.

and referring to postwar cross-country evidence from Kuznets (1973) they conclude that the theory faces a *population puzzle*.

There are several attempts of an explanation of the population puzzle which should be mentioned. Jones (1995, pp. 777–778) in his seminal paper on semi-endogenous growth claims that his model was not contradicted by the evidence since it should only be applied to advanced economies or even to a part of the whole world, and since in his approach it was not population but the number of researchers that matters for growth. There is not much to disagree with his arguments which, however, after all imply that his model does not address the implications of population growth for international differences in the growth rates of per capita incomes. Moreover, they presuppose that the number of researchers does not rise with the level of population. The arguments of Goodfriend and McDermott (1995, pp. 128–129) are similar in that they emphasize that human capital per capita rather than population itself accounted for the diversity of per capita product in industrialized countries.

The argument that the relevant unit of analysis was a part of the whole world obviously relies on the presence of knowledge spillovers across national borders. Recent empirical evidence by Branstetter (2001), however, suggests that knowledge spillovers are primarily intranational and not international in scope.[3] It is therefore reasonable to consider the implications of population growth for nationally localized technical progress in order to get an understanding of differences in the growth and development of nations as possibly caused by differences in population growth rates.

The population puzzle need not arise in models of endogenous population growth (cf. e.g. Becker et al., 1990; Galor and Weil, 2000).[4] The present paper provides a complementary argument by showing that even an exogenously rising population growth rate may imply a declining growth rate of per capita income if international trade is taken into consideration. While the literature on economic growth in open economies usually concentrates on the relation between openness and per capita income growth and convergence (cf. e.g. Ben-David and Loewy, 1998), it is also important to consider the implications of different population growth rates for the development of open economies. Such an investigation is pursued here using a model in which international trade is a precondition for sustained growth given the available technology in the home country, which needs imported intermediate products to produce its output. As a reasonable consideration of positive rates of population growth is not possible in models involving scale effects, non-scale growth theory lends itself to this task.

[3]As Feenstra (1996, p. 231) remarks, various negative tests of the predictions of the Heckscher-Ohlin model of international trade also provide compelling evidence of substantial lags in the international flow of technical knowledge.

[4]The model of Galor and Weil (2000) describes the evolution of population, human capital, technology, and output starting at a Malthusian regime through a Post-Malthusian regime to a Modern Growth regime. While the population puzzle does not arise with respect to the comparison of different regimes, it still shows up with respect to differences in population growth rates in the Modern Growth regime.

More specifically, a framework with imported intermediate goods formulated more than 30 years ago by Khang (1968) and Bardhan (1970, ch. 4) will be augmented by a non-scale growth approach of the learning by doing type which is a special case of the two-sector model in Eicher and Turnovsky (1999). Analytical tractability follows from an exogenous growth rate of the rest of the world and its demand for the domestic commodity. Even if a general equilibrium formulation including the rest of the world may be desirable, the model is plausible in that it captures the idea of *demand limitation*, which is still much of a problem for many developing countries (DCs). The income elasticities of export demand from DCs are generally smaller than for goods from industrialized countries (ICs) (cf. e.g. Senhadji and Montenegro, 1999), although for most DCs they nevertheless exceed one (particularly in Asia). It will become apparent that a demand limitation will constrain the growth of DCs even if export demand is not income-inelastic. E.g., if the product of the income elasticity with the growth rate of GDP in ICs falls short of the rate of population growth in the DCs and there is no technical progress, consumption and income per capita in the DCs will decline. A similar though more complicated proposition applies in the presence of technical progress. As the data on average growth rates from 1980 to 1999 reported by the World Bank (World Development Indicators 2001, Tables 2.1 and 4.1) show, the population growth rates of some DCs actually exceeded the growth rates of GDP in major ICs. It will be shown among other things that, in combination with endogenous technical progress, the demand limitation implies that consumption per capita may decline or increase in the rate of population growth. The result depends on the relative magnitudes of particular production, learning, and demand elasticities. Thus, the model reveals that a higher rate of population growth may have favorable as well as unfavorable effects on economic development, depending on the specific situation. E.g., for a small open economy, which is included as a special case of the model, a higher growth rate of population has the usual positive effects on the growth rate of per capita income.

Introducing an exogenous world export demand into a supply-driven non-scale growth model amounts to combining a neoclassical growth model with a Keynesian demand limitation in an open economy. In fact, the export demand equation that will be used is similar to the one used by Thirlwall (1979) in deriving and testing what has become known as *Thirlwall's Law* (namely, the long-run growth rate of domestic income is constrained by the ratio of the growth rate of world export demand to the domestic income elasticity of demand for imports). It will be seen that this assertion does not hold in the present model, although the growth rate of exports shows up as one of the determinants of the growth rate of (per capita) output. Thus, the balance of payments constrains the growth rate in this otherwise neoclassical model, which thereby also contributes to an integration of neoclassical and Keynesian aspects of economic growth.

The remainder of the paper is organized as follows. In Section 2, a model of non-scale growth with imported inputs is analyzed. It is shown that under reasonable assumptions a globally asymptotically stable steady state exists and the implications for long-run growth are derived with respect to this steady state. The final section discusses the main assumptions and results as well as their policy implications. The lengthier calculations are relegated to appendices, where the case of R&D-driven growth is also briefly discussed.

2 A Model of Non-Scale Growth with Imported Inputs

2.1 The Model and Its Static Equilibrium

As in the seminal paper on the economic implications of learning by doing (LBD) by Arrow (1962), it is usual in growth models to measure the *learning index* by the integral of past gross investment. In the present paper, the learning index will be measured by cumulated production, which is more in line with the empirical evidence on LBD. Especially the empirical results of Lieberman (1984) substantiate the following assumptions of the model: LBD is a major source of technical progress involving a high degree of externalities (knowledge spillovers). As a simplified approximation to reality, it is assumed that LBD is the only source of technical progress and is purely external at the firm level. This assumption enables a setting of perfect competition. Knowledge spillovers are only national, not international in scope, however (as suggested by the study of Branstetter, 2001). The learning elasticity (the parameter β in equation (1) below) is assumed to be positive but much smaller than one (empirically estimated values tend to be around 0.3). Finally, the learning function enters the production function in a multiplicative way, that is, technical progress is Hicks-neutral. While this assumption has the disadvantage that a steady state does not exist unless the production function is of the Cobb-Douglas type (which implies the equivalence of Hicks-neutrality and Harrod-neutrality), it appears to be more in line with the empirical evidence than the usually assumed Harrod-neutral form. Although one could of course argue that the model is actually one with Harrod-neutrality, I consider it as a simple approximation to other functional forms in which the equivalence does not hold, however. It should also be noted that the results of Eicher and Turnovsky (1999, p. 413) suggest that even in more general models than the one considered here a non-scale balanced growth path with increasing per capita income will only exist in the Cobb-Douglas case or an alternative separability condition which does not add much more generality.

Hence, denoting the aggregate output of the home economy by X, the production function is assumed to be of the Cobb-Douglas type:

$$X = A^{\beta} K^{\alpha_1} M^{\alpha_2} L^{1-\alpha_1-\alpha_2}, \quad 0 < \alpha_1, \alpha_2 < 1,\ 1-\alpha_1-\alpha_2 > 0,\ \beta > 0. \qquad (1)$$

Producers in the home economy act in a setting of perfect competition. There are three inputs, physical capital, K, an imported intermediate product, M, and labor, L. The variable A denotes the learning index. Of course, all variables depend on time, but for the sake of notational convenience the time index t has been dropped.

In an analogous closed economy model with $\alpha_2 = 0$ the condition $1 - \alpha_1 - \beta > 0$ would be necessary for the existence of a steady state path with positive growth rates of output and population (cf. Appendix A). It could be guessed that with respect to the model at hand the condition $1-\alpha_1-\alpha_2-\beta > 0$ must be met, and this assumption would indeed accord well with empirical evidence. As has been noted before, β should be expected to be positive but much smaller than one, and since $1 - \alpha_1 - \alpha_2$ equals the share of labor, it should exceed this value of β sufficiently to guarantee that $1 - \alpha_1 - \alpha_2 - \beta > 0$. As it turns out, however, this condition is more restrictive than needed [cf. relation (18) and Appendix A].

The importance of intermediate and capital goods imports from industrialized countries (ICs) for developing countries (DCs), on which the model due to the assumption $\alpha_2 > 0$ heavily relies, is underlined by a well known study of Havrylyshyn and Civan (1985) on intra-industry trade among DCs. In particular with respect to the newly industrialized countries (NICs) it has been found that only 5 percent of the imports of investment goods by NICs come from other NICs. Thus, ICs clearly "outcompete NICs in sales of machineries to NICs" (p. 267), and the "global comparative advantage of NICs is probably still largely in labor-intensive consumer goods." It is therefore reasonable to model the trade relations by the assumption that DCs import intermediate goods from ICs in exchange for consumer goods as in Khang (1968) and Bardhan (1970). To keep the analysis as simple as possible, imported capital goods will be neglected.

Thus, in order to produce its output using the technology specified in (1), the home country must import the amount M of the intermediate product from the rest of the world, henceforth referred to as the foreign country. Neglecting international borrowing and lending and denoting the amount of domestic output exported by EX, the balance of trade equation

$$pEX = M \tag{2}$$

must be satisfied at every instant, where p is the price of domestic output in terms of the intermediate good, which is taken as the numéraire.[5] Since it is assumed that free trade prevails, p equals the terms of trade of the home country. As in Bardhan (1970, ch. 4), it is assumed that world export demand for the domestic commodity is given by

$$EX = p^{\eta} e^{\lambda t}, \quad \eta < 0,\ \lambda > 0. \tag{3}$$

Notice that world export demand would grow at the rate λ if the terms of trade were constant. Thus, λ will be referred to as the *growth rate of world export demand at fixed prices.*

[5] In reality, it is almost never the case that the balance of trade always equals zero. Likewise, however, there are no examples of countries accumulating external debt at an exponential rate in the long run. Thus, balanced trade appears to be a natural assumption in a model of long-run growth. It is shown in Appendix D that the principal results do not depend on this assumption, however, even if external debt was allowed to grow at an exponential rate.

Since world export demand is of special importance for the analysis, the underlying preferences will briefly be analyzed. Suppose that $e^{\lambda t} = \mu_1 Y^{*\mu_2}$, where Y^* is foreign income in terms of the intermediate good and $\mu_1 > 0$, $\mu_2 > 0$ are parameters. If Y^* grows at the constant rate $\bar{\lambda}$, $Y^* = Y_0^* e^{\bar{\lambda} t}$, it must be that $e^{\lambda t} = \mu_1 Y_0^{*\mu_2} e^{\mu_2 \bar{\lambda} t}$, with normalization according to $\mu_1 Y_0^{*\mu_2} = 1$ and $\lambda := \mu_2 \bar{\lambda}$. In other words, equation (3) can be interpreted as resulting from a demand function

$$EX = \mu_1 p^{\eta} Y^{*\mu_2} \tag{4}$$

with constant price and income elasticities η and μ_2, respectively, where Y^* grows at the constant rate $\bar{\lambda} = \lambda/\mu_2$. Assuming that households in the foreign country consume two goods and solving the integrability equation (cf. e.g. Varian, 1992, p. 127) yields different indirect utility functions depending on the values of η and μ_2. If $\mu_2 = 1$, the underlying preferences are homothetic including the special case of Cobb-Douglas preferences for $\eta = -1$. If $\mu_2 \neq 1$, preferences are non-homothetic. These results are important because it will be shown that some implications of the model depend on the size of λ, which in turn depends on the homotheticity of foreign preferences and the growth rate of foreign income.

National income (value added) in terms of the imported intermediate product is

$$Y = pX - M. \tag{5}$$

In a setting of perfect competition, the marginal value product of M equals its price, 1, which implies

$$M = p\alpha_2 X. \tag{6}$$

Substitution into (5) yields

$$Y = p(1 - \alpha_2)X. \tag{7}$$

To obtain the static equilibrium solution of the model, the export demand (3) is substituted into the balance of trade equation (2), which is solved for

$$\begin{aligned} p &= M^{\frac{1}{\eta+1}} e^{-\frac{\lambda}{\eta+1} t} \quad \text{if} \quad \eta \neq -1, \\ M &= e^{\lambda t} \quad \text{if} \quad \eta = -1. \end{aligned} \tag{8}$$

Substitution into (6) and using (1) if $\eta \neq -1$ yields

$$\begin{aligned} M &= \alpha_2^{\frac{\eta+1}{\eta-\alpha_2(\eta+1)}} A^{\frac{\beta(\eta+1)}{\eta-\alpha_2(\eta+1)}} K^{\frac{\alpha_1(\eta+1)}{\eta-\alpha_2(\eta+1)}} L^{\frac{(1-\alpha_1-\alpha_2)(\eta+1)}{\eta-\alpha_2(\eta+1)}} e^{-\frac{\lambda}{\eta-\alpha_2(\eta+1)} t} \quad \text{if } \eta \neq -1, \\ p &= \frac{1}{\alpha_2 X} e^{\lambda t} \quad \text{if } \eta = -1. \end{aligned}$$

The respective values of M can be inserted into (1) to obtain

$$X = BA^{\beta\varphi} K^{\alpha_1 \varphi} L^{(1-\alpha_1-\alpha_2)\varphi} e^{-(\lambda\alpha_2\varphi/\eta)t}, \tag{9}$$

which is valid regardless whether $\eta \neq -1$ or $\eta = -1$, and where

$$\varphi := \frac{\eta}{\eta - \alpha_2(\eta+1)} \quad \text{and} \quad B := \alpha_2^{\frac{\alpha_2(\eta+1)}{\eta-\alpha_2(\eta+1)}}. \tag{10}$$

Notice that if $\eta = -1$, then $\varphi = B = 1$. Equation (9) is the basic relation of the model summarizing the static equilibrium solution for output X as a function of variables which are predetermined at any moment. The denominator in the expression for φ is negative since $\eta < 0$, and

$$B > 0, \quad \varphi > 0, \quad \varphi \lesseqqgtr 1 \quad \Longleftrightarrow \quad \eta \gtreqqless -1.$$

2.2 Dynamics and Stability

The population equals the labor force and grows at an exogenous and constant rate $n > 0$:

$$g_L := \frac{\dot{L}}{L} = n.$$

In general, the growth rate of a variable x is written as g_x. The learning index, A, is defined as cumulative production. Thus, the time derivative of A is

$$\dot{A} = X = A^\beta K^{\alpha_1} M^{\alpha_2} L^{1-\alpha_1-\alpha_2}. \tag{11}$$

Output can both be consumed and invested. Domestic households decide about consumption, C, and gross savings, S, which in short-run equilibrium equal gross investment, I. If a fixed fraction, s, of national income in terms of the domestic commodity, Y/p, is saved ($pS = sY$), consumption will be

$$C = (1-s)Y/p = (1-s)(1-\alpha_2)X. \tag{12}$$

More generally, aggregate gross investment is given by the difference of output, X, and the sum of consumption and exports, $C + EX$, all measured in terms of the domestic commodity. Neglecting the depreciation of capital for simplicity, gross investment equals net investment and the time derivative of the capital stock is

$$\dot{K} = I = X - EX - C = \frac{Y}{p} - C = (1-\alpha_2)X - C, \tag{13}$$

where (2), (5), and (7) have been used. Substituting (12) in case of a fixed saving rate yields

$$\dot{K} = s(1-\alpha_2)X. \tag{14}$$

Although a fixed saving rate will be assumed in the sequel, the principal conclusions do not depend on this assumption. E.g., it is straightforward to show that the major implications of the model concerning the long-run development of the economy in a steady state would remain valid if either a classical savings function or a decentralized optimizing framework was employed.

A steady state is defined as a growth path along which all variables grow at constant rates. If $g_A = \dot{A}/A$ is constant, it follows that $\dot{A}/A = \ddot{A}/\dot{A}$ and therefore according to equation (11) that $g_A = g_X$. Also, if $g_K = \dot{K}/K$ is constant, equations (14) and (12) imply that $g_K = g_C = g_X$. It is shown in Appendix A that the following growth rates apply in a steady state:

$$\begin{aligned} g_X = g_C = g_K = g_A &= \underbrace{\frac{-(1-\alpha_1-\alpha_2)\eta}{\alpha_2-(1-\alpha_1-\alpha_2-\beta)\eta}}_{=:\gamma_1>0} n + \underbrace{\frac{\alpha_2}{\alpha_2-(1-\alpha_1-\alpha_2-\beta)\eta}}_{=:\gamma_2>0} \lambda \\ &= \gamma_1 n + \gamma_2 \lambda, \qquad (15) \\ g_p &= \frac{1}{\eta} g_X - \frac{\lambda}{\eta}, \qquad (16) \end{aligned}$$

and (7) immediately implies

$$g_Y = g_p + g_X. \qquad (17)$$

It is also shown in the appendix that

$$1 - \alpha_1 - \alpha_2 - \beta > \alpha_2/\eta, \quad \text{or} \quad \alpha_2 - (1-\alpha_1-\alpha_2-\beta)\eta > 0 \qquad (18)$$

is necessary for the existence of a steady state with positive growth rates of output and population, which will be assumed. Thus, the denominator in γ_1 and γ_2 is positive. Hence, since it has been supposed in equations (1) and (3) that $1-\alpha_1-\alpha_2 > 0$, $\alpha_2 > 0$, and $\eta < 0$, both γ_1 and γ_2 are positive. Equations (15)–(17) cover the central implications of the model and are comprehensively analyzed in Section 2.3.

In a steady state, $g_X - \gamma_1 n - \gamma_2 \lambda = 0$, and the same applies to g_A, g_K, and g_C. Thus, the following *scale adjusted* per capita variables are constant in long-run equilibrium:

$$x := \frac{X}{L^{\gamma_1} e^{\gamma_2 \lambda t}}, \; a := \frac{A}{L^{\gamma_1} e^{\gamma_2 \lambda t}}, \; k := \frac{K}{L^{\gamma_1} e^{\gamma_2 \lambda t}}, \; c := \frac{C}{L^{\gamma_1} e^{\gamma_2 \lambda t}}. \qquad (19)$$

It is therefore natural to express equation (9) in terms of these scale adjusted per capita variables (cf. Appendix B):

$$x = B a^{\beta\varphi} k^{\alpha_1 \varphi}. \qquad (20)$$

It is also shown in Appendix B that the model can now be reduced to a system of two differential equations in k and a:

$$\dot{k} = s(1-\alpha_2) B a^{\beta\varphi} k^{\alpha_1\varphi} - (\gamma_1 n + \gamma_2 \lambda) k, \qquad (21)$$
$$\dot{a} = B a^{\beta\varphi} k^{\alpha_1\varphi} - (\gamma_1 n + \gamma_2 \lambda) a. \qquad (22)$$

Figure 1 shows the phase portrait of the dynamical system (21), (22). It is proven in Appendix C that the isoclines $\dot{k} = 0$ and $\dot{a} = 0$ have the shape depicted in the figure and that a unique equilibrium $E = (\bar{k}, \bar{a})$ in the positive orthant exists. As

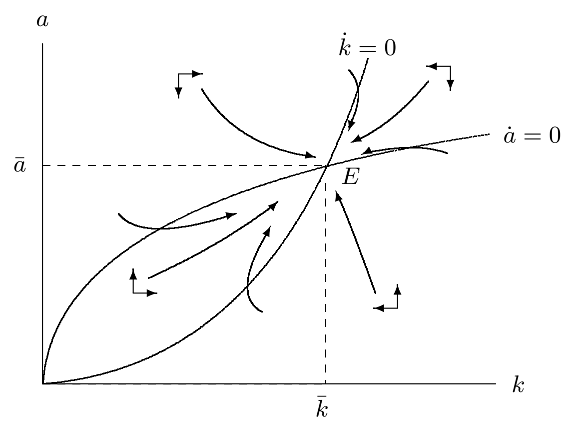

Figure 1. Global Stability of the System (21), (22)

can be seen from the phase diagram, the equilibrium E is globally asymptotically stable for all strictly positive initial values.[6] Although the transitional dynamics to the steady state also has interesting implications, stability justifies to center the analysis in the next section around the properties of the long-run equilibrium.

2.3 Properties of the Steady State

This section presents the major propositions about the steady state of the model economy. All propositions assume that the assumptions about the parameters stated in equations (1), (3), and (18) are met. Therefore, these assumptions will not be repeated each time a proposition is stated.

Consider at first the special case in which no technical progress occurs ($\beta = 0$). The state variable a then drops out and the model reduces to Bardhan's (1970, ch. 4) contribution, in which $\gamma_1 + \gamma_2 = 1$. Thus, in this case g_X is a weighted average of the growth rates of population, n, and world demand for the domestic product at fixed prices, λ. If $\lambda < n$, output X therefore would grow at a rate smaller than n and per capita output would steadily decline. As it follows from Proposition 1 below that the terms of trade, p, also decline under these circumstances, national income per capita a fortiori decreases steadily. Bardhan (1970, p. 68, fn. 4) attributes this property to the export "demand limitation on growth" and he states that such a situation could not go on forever in the real world, where offsetting factors as technical progress and foreign aid would exist. It will be seen, however, that technical progress need not hinder such a steady decline of per capita income (cf. Proposition 3).

In the general case, inserting (15) into (16) and rearranging yields the following

[6]It should be noted that no mathematical proof of global asymptotical stability is necessary in case of a phase portrait as shown in Figure 1, which reveals the desired property according to the arguments set forth in Hirsch and Smale (1974, ch. 12).

condition determining the growth rate of the terms of trade, p:

$$g_p = \frac{(1-\alpha_1-\alpha_2-\beta)\lambda - (1-\alpha_1-\alpha_2)n}{\alpha_2 - (1-\alpha_1-\alpha_2-\beta)\eta}. \tag{23}$$

Notice that $1-\alpha_1-\alpha_2-\beta$ is the difference between the production elasticity of labor (the labor-share under perfect competition) and the learning elasticity. Since the denominator in (23) is positive, this equation immediately implies

Proposition 1 *If the growth rate of world export demand at fixed prices, λ, falls short of the growth rate of population in the home country, n, its terms of trade will worsen steadily ($g_p < 0$). The terms of trade will improve if and only if $\lambda > n$ and λ weighted by the difference between the labor-share and the learning elasticity exceeds n weighted by the labor-share.*

Notice that for the special case $\beta = 0$ the terms of trade decrease, are constant or increase according to whether λ falls short of, equals or exceeds n.

According to (23), g_p is zero if $(1-\alpha_1-\alpha_2-\beta)\lambda = (1-\alpha_1-\alpha_2)n$, in which case (15) and (17) imply that $g_Y = g_X = \lambda$. Since λ is the growth rate of exports at constant prices and the domestic income elasticity of imports equals one [which can be seen by substituting for X in (6) by (5) and solving for M], this result is a version of Thirlwall's Law mentioned in Section 1 (which also assumes $g_p = 0$). As can be seen from (23), however, this result requires a theoretically unlikely knife-edge condition.[7] In the more general case considered here, g_Y is influenced by the supply as well as the demand side. It is interesting to notice that according to equation (15) the importance of the balance of payments constraint rises as the price elasticity η is getting smaller in absolute value. If $\eta \to 0$, then $g_X = g_C = \lambda$, which is the Keynesian case in which the growth rate of export demand determines the growth rate of output and consumption at home, although g_p will generally be different from zero and therefore $g_Y \neq g_X$. If $\eta \to -\infty$, then $g_p \to 0$ (without any knife-edge conditions) and applying de l'Hôspital's rule to (15) yields

$$g_Y = g_X = g_C = \frac{1-\alpha_1-\alpha_2}{1-\alpha_1-\alpha_2-\beta} n = \gamma_1 n, \tag{24}$$

which corresponds to the neoclassical small country case where export demand does not constrain the growth rates at home.[8] Thus, the Keynesian case does not arise if $g_p = 0$, but if $\eta \to 0$, while Thirlwall's assumption $g_p = 0$ in the present model requires $\eta \to -\infty$ (except for a knife-edge-case) and therefore gives rise to neoclassical results.

[7] As to the empirical problems involved in assessing the long-run evolution of the developing countries' terms of trade, cf. Powell (1991) and the literature cited therein.

[8] To be particular about the small country case, equation (3) has to be substituted by the assumption of an exogenous relative world price, $\bar{p}$. Using $\bar{p}$ in (6) and substituting into (1) yields the small country version of (9). The steady state growth rate implied by this equation coincides with the one given in (24), however.

Next consider the growth rate of consumption C, which equals the growth rates of output, capital, and the learning index as provided in (15). Using the definitions of γ_1 and γ_2 in $g_{C/L} = \gamma_1 n + \gamma_2 \lambda - n$ leads to

$$g_{C/L} = \frac{\alpha_2 \lambda - (\beta\eta + \alpha_2)n}{\alpha_2 - (1 - \alpha_1 - \alpha_2 - \beta)\eta}. \tag{25}$$

It follows that

$$g_{C/L} \gtreqless 0 \quad \Longleftrightarrow \quad \lambda \gtreqless \frac{\alpha_2 + \beta\eta}{\alpha_2} n.$$

Thus, consumption per capita would rise only if the growth rate of world export demand at fixed prices, λ, exceeded the growth rate of the domestic population, n, weighted by $(\alpha_2 + \beta\eta)/\alpha_2$. Positive per capita growth is the more likely, the greater are λ, the absolute value of the negative price elasticity, $|\eta|$, and the learning elasticity, β, and the smaller are n and the production elasticity of imported intermediates, α_2. Equation (25) implies that in case of technical progress ($\beta > 0$) and a world export demand growing at least as fast as the domestic population ($\lambda \geqq n$), per capita consumption at home grows at a positive rate. If world export demand grew at a rate smaller than the growth rate of domestic population, however, it would be possible that per capita consumption declines. More specifically, $g_{C/L} < 0$ if $\alpha_2 > -\beta\eta$ and n exceeded the threshold value $\bar{n} := \alpha_2\lambda/(\alpha_2 + \beta\eta) > \lambda$. It is important to observe, however, that it may take a really high value of n to exceed $\bar{n}$ because the denominator $\alpha_2 + \beta\eta$ must be positive and should therefore be expected to be a small number. Thus, a negative growth of per capita consumption will occur only if there is a low rate of technical progress and n largely exceeds λ.

Now consider the derivative

$$\frac{\partial g_{C/L}}{\partial n} = \frac{-(\beta\eta + \alpha_2)}{\alpha_2 - (1 - \alpha_1 - \alpha_2 - \beta)\eta} \gtreqless 0 \quad \Longleftrightarrow \quad \alpha_2 \lesseqgtr -\beta\eta.$$

This relation together with the foregoing analysis provides the following solution of the *population puzzle*:

Proposition 2 *The growth rate of consumption per capita, $g_{C/L}$, increases, is constant, or decreases in the rate of population growth depending on whether the production elasticity of the imported intermediate good falls short, equals, or exceeds the learning elasticity weighted by the absolute value of the price elasticity of demand for exports. If $g_{C/L}$ decreases in n, there will be a threshold value $\bar{n} := \alpha_2\lambda/(\alpha_2 + \beta\eta)$ such that $g_{C/L} \gtreqless 0$ if $n \lesseqgtr \bar{n}$. If $g_{C/L}$ increases in n, it will be positive.*

Since $g_X = g_C$ in a steady state, Proposition 2 applies as well with respect to $g_{X/L}$.

Hence, the greater the production elasticity of imported intermediates, the less price-elastic world export demand, and the smaller the learning elasticity, the more likely will a higher rate of population growth imply a lower rate of growth of per capita consumption. These results are fairly intuitive. As the empirical evidence of

Havrylyshyn and Civan (1985) suggests, developing countries import intermediates largely from industrialized countries (α_2 matters) and export goods with relatively low capital intensities, which according to the results of Lieberman (1984) entail relatively low learning elasticities (β). The relative magnitudes of these parameters probably reverse with a rising degree of development. At least if the home country in the model economy is interpreted as representing a group of similar countries in reality, an expansion of sales will require considerably lower prices (small $|\eta|$). Indeed, the findings of Senhadji and Montenegro (1999) show that export price elasticities are relatively low ($|\eta| \approx 1$ on average) even with respect to individual countries. Proposition 2 therefore provides an explanation why per capita growth rates sometimes rise and sometimes fall with the rate of population growth.

The economic intuition underlying Proposition 2 can be seen most easily by considering the special case $\eta = -1$. Equation (9) then reads

$$X = A^{\beta} K^{\alpha_1} L^{1-\alpha_1-\alpha_2} e^{\alpha_2 \lambda t},$$

where $M = e^{\lambda t}$ according to (8) grows at an exogenous rate which is independent of the development of p. Whether $g_{C/L}$ increases (is constant, decreases) in n according to Proposition 2 now depends on whether $\beta > \alpha_2$ ($\beta = \alpha_2$, $\beta < \alpha_2$). This condition determines whether there are increasing (constant, decreasing) returns to scale with respect to K, A and L. As $M = e^{\lambda t}$ is independent of n while the steady state growth rates of K, A and L increase in n, it follows that the per capita growth rate rises (is constant, falls) in n if there are increasing (constant, decreasing) returns with respect to those inputs which grow depending on n in the steady state.

It should finally be noted that the empirical evidence does not refer to consumption but income (GDP) per capita. While output X unambiguously increases according to (15), the growth rate g_Y of national income measured in terms of the intermediate produced abroad may be negative due to a falling relative price of the domestic commodity. It follows that income per capita, Y/L, may grow at a negative rate all the more. Since $g_{Y/L} = g_p + g_X - n = g_p + g_{C/L}$, adding the growth rates in (23) and (25) yields

$$g_{Y/L} = \frac{(1-\alpha_1-\beta)\lambda - (1-\alpha_1+\eta\beta)n}{\alpha_2 - (1-\alpha_1-\alpha_2-\beta)\eta}. \tag{26}$$

In the special case $\lambda = n$ and $\eta = -1$ or $\beta = 0$, income per capita is constant. Without technical progress ($\beta = 0$), income per capita decreases, is constant, or increases according to whether $\lambda \lesseqqgtr n$.

The signs of the coefficients in the nominator of (26) are restricted as follows. Adding $1 - \alpha_1$ to both sides of the second inequality in (18) and rearranging leads to

$$1 - \alpha_1 + \eta\beta > (1+\eta)(1-\alpha_1-\alpha_2).$$

If world export demand is price-inelastic ($\eta > -1$), the right hand side is positive. Thus, $1 - \alpha_1 + \eta\beta > 0$ if $\eta > -1$. Similarly, it can be shown using (18) that

$1-\alpha_1-\beta>0$ if $\eta \leqq -1$. It follows from (26) that

$$
\begin{aligned}
g_{Y/L} \gtreqqless 0 \quad &\Longleftrightarrow \quad n \lesseqqgtr \frac{1-\alpha_1-\beta}{1-\alpha_1+\eta\beta}\lambda \quad \text{if} \quad \eta > -1, \\
g_{Y/L} \gtreqqless 0 \quad &\Longleftrightarrow \quad \lambda \gtreqqless \frac{1-\alpha_1+\eta\beta}{1-\alpha_1-\beta}n \quad \text{if} \quad \eta \leqq -1.
\end{aligned} \tag{27}
$$

In both cases, the fraction on the right hand side of the respective second inequality is smaller than one, and while both denominators are positive, the respective nominators may even be negative. Differentiating $g_{Y/L}$ with respect to n yields

$$\frac{\partial g_{Y/L}}{\partial n} \gtreqqless 0 \quad \Longleftrightarrow \quad 1-\alpha_1+\eta\beta \lesseqqgtr 0.$$

This relation, (27), and the foregoing discussion imply the following

Proposition 3 *If world export demand is price-inelastic ($\eta > -1$), the per capita income growth rate, $g_{Y/L}$, will fall in the growth rate of population, n. In this case, $g_{Y/L}$ will be positive only if $1-\alpha_1-\beta>0$ and λ exceeds n by at least the factor $(1-\alpha_1+\eta\beta)/(1-\alpha_1-\beta)>1$. If export demand is price-elastic ($\eta \leqq -1$), $g_{Y/L}$ may be positive even if $\lambda < n$. In this case, $g_{Y/L}$ rises, is constant, or falls in n according to whether $1-\alpha_1+\eta\beta$ is negative, zero, or positive.*

Proposition 3 confirms the solution of the *population puzzle* with respect to income per capita measured in terms of the imported intermediate product.

Finally, it is also instructive to consider $g_{Y/L}$ as a function of the learning elasticity, β. Differentiating (26) with respect to β yields, after a tedious but straightforward calculation,

$$\frac{\partial g_{Y/L}}{\partial \beta} \gtreqqless 0 \iff (1+\eta)\eta(1-\alpha_1-\alpha_2)n \gtreqqless (1+\eta)\alpha_2\lambda \iff 1+\eta \lesseqqgtr 0.$$

In other words, the growth rate of per capita income in the home country will rise with the degree of technical progress only if world demand for the exported commodity is price-elastic. In case of a price-inelastic demand, a kind of *dynamic immiserizing growth* with respect to income per capita occurs. Speeding up technical progress leads to a decline in the growth rate of per capita income.

3 Concluding Remarks

The results of this model differ substantially from the implications of an analogous non-scale growth closed economy model, where all growth rates rise in the growth rate of population. These differences arise since world export demand constrains the availability of intermediate inputs and therefore the growth rates of output and national income. While it would be desirable to endogenously explain world export demand, the principal conclusions drawn from this model of balance of payments constrained non-scale growth are nevertheless convincing. In Sections 1 and 2.1, respectively, empirical evidence about the growth rates of world export demand and population and the importance of intermediate goods imports for developing countries (DCs) has been cited. This evidence underlines the plausibility of this theory about the effects of population growth rates on the growth of per capita incomes. The *population puzzle* can thus be solved using a simple and analytically tractable model that emphasizes that a higher rate of population growth may have favorable as well as unfavorable effects on economic development, depending on the relative magnitudes of various elasticities. Due to the underlying assumptions the present model is mainly applicable to developing countries, for which the population puzzle is most apparent. It is shown in Appendix E, however, that a similar explanation is also feasible with respect to industrialized countries, although the parameter values necessary for a negative relationship between population growth and the growth of per capita income are less likely to be met in this case.

As to the specified form of the exogenous world export demand function, it is important to pay attention to an appropriate interpretation of the results. The theory of international trade considers different levels of international interaction, at the one extreme the single small country in a big world with all prices given, at the other extreme a two country world with both countries being large enough to influence world prices. While in mainstream trade theory the countries differ only with respect to the magnitude of some parameters, the so-called *North-South models* supplement these approaches by introducing structural differences between the countries. With respect to the present model the home country may either be interpreted as a single country or as a group of similar countries of the South facing a given export demand of the North, which may be price-inelastic and does not grow at an infinite rate. The importance of considering different degrees of international interaction is also demonstrated by the sensitivity of the present model with respect to the assumption about the price elasticity of export demand, η. The small country assumption is included as a special case of the model by letting $\eta \to -\infty$. Equation (24) then implies that the growth rate of per capita consumption is $g_{C/L} = (\gamma_1 - 1)n > 0$, which basically coincides with an analogous neoclassical closed economy model where $g_{C/L}$ increases in the population growth rate, n. If $\eta \to 0$, however, the export demand limitation is getting more important, similar to Keynesian models of balance of payments constrained growth.

With respect to those implications of the model which depend on the relative magnitude of the growth rate of world demand at fixed prices, λ, it is important to recognize that there are various reasons why λ could fall short of n, e.g. As has been shown in Section 2.1, λ has to be interpreted as the product of the growth rate of foreign income, $\bar{\lambda}$, and the income elasticity, μ_2. E.g., if $\lambda < n$, the domestic terms of trade will worsen steadily, no matter whether λ is relatively low because of a relatively low growth rate abroad or because the domestic product is an inferior commodity ($\mu_2 < 1$). Similarly, the domestic economy profits from a high growth rate abroad in the same way as it profits from a high income elasticity with respect to its exported product.

While some of the implications of the model depend on the relative magnitudes of the rate of population growth, n, and the growth rate of world export demand at fixed prices, λ, it is important to notice that the first part of Proposition 2 is independent of λ. Thus, even if world export demand grew much faster than the domestic population, a higher rate of population growth would imply a lower growth rate of per capita consumption if the production elasticity of intermediates exceeded the learning elasticity weighted by the absolute value of the price elasticity of export demand, $|\eta|$. This condition could not be met in the small country case with an infinitely elastic demand and a positive learning elasticity. While empirically estimated demand elasticities are usually relatively low (e.g., according to Senhadji and Montenegro, 1999, $|\eta| \approx 1$), Panagariya et al. (2001) recently have found a value of 26 for multi-fibre products imported from Bangladesh, a value in favor of "trade economists' intuition" about small countries. As these authors remark themselves, however, other estimates are usually much lower. Whichever estimates match the true values better, they generally refer to single countries. At least if the exports of several countries of the south grow at the same time, the effect of these countries' exports together on the terms of trade have to be taken into account. With respect to such a group of similar countries, the demand elasticities should be relatively low. The model therefore provides a reasonable prediction about the negative influence of population growth rates on the growth of per capita consumption in some DCs.

Similar remarks apply to the effect of population growth rates on the growth rates or per capita income in terms of the intermediate product. The importance of the demand elasticity is even more apparent. Whenever world demand is price-inelastic, per capita income growth rates will fall in n, and if n exceeds a particular threshold value under these circumstances, the growth rate will even be negative. It should be kept in mind that this prediction does only apply to those countries which are not already in a position to produce the intermediate goods themselves.

This observation leads to the policy implications of the analysis. Here, two different approaches must be distinguished. The first is to look for an optimum policy mix *given* the current state of the production structure and the growth rate of population. The model could be augmented by introducing a government raising tariffs in order to influence the terms of trade in favor of the domestic economy and granting subsidies to possibly increase the effects of LBD. It would be necessary, however, to analyze whether capital accumulation should really be increased since it does not only have a positive effect on LBD but also an adverse effect on the terms of trade through increased production. Such an analysis is not pursued here, however, since this first type of policy does not address the core of the problem.

What really matters is the second type of policy, which does *not* take the current production structure as given but aims at a structural change. Industrialized countries (ICs) have production technologies at their disposal which DCs have not. Apart from birth controls, the only way out of a dilemma with a high growth rate of population and a low growth rate of per capita consumption for the DCs is trying to imitate the production technologies of the ICs with respect to intermediate products and thereby eliminating the foreign demand constraint on growth. Among the arguments in favor of free trade respectively an outward-looking development policy in the literature, one of the most convincing is that international trade can act as an impetus for the flow of knowledge across international borders (cf. e.g. Ben-David and Loewy, 1998, for a model along such lines). It is the present author's conviction that this is a reasonable suggestion, which however should not be taken to imply that laissez faire alone could solve all problems. As has already been noted, the empirical findings of Branstetter (2001) suggest that knowledge spillovers are primarily intranational, not international in scope. Of course, this evidence does not imply that no international spillovers at all exist. It is obvious that international trade will at least transfer the knowledge about the existence of goods which are not available at home. The flow of such basic knowledge, however, would only be prevented in case of prohibitive protectionism. Thus, John S. Mill had a point arguing that after the initial adaption of a foreign production technology a temporary protection of the infant industry could be necessary to gain sufficient familiarity with the new technology.

Colophon

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Appendix

A Steady State Growth Rates

The assumptions about the parameters stated in equations (1) and (3) are assumed to hold. Logarithmic differentiation of equation (9) implies

$$g_X = \beta\varphi g_A + \alpha_1\varphi g_K + (1-\alpha_1-\alpha_2)\varphi g_L - \alpha_2\varphi\lambda/\eta.$$

Using $g_L = n$ and $g_K = g_A = g_X$ in a steady state implies

$$(1-\alpha_1\varphi-\beta\varphi)g_X = (1-\alpha_1-\alpha_2)\varphi n - \alpha_2\varphi\lambda/\eta. \tag{A1}$$

Since the right hand side will be positive if $n > 0$ and/or $\lambda > 0$, the existence of a steady state with $g_X > 0$ requires that $1-\alpha_1\varphi-\beta\varphi > 0$. (The condition $1-\alpha_1\varphi-\beta\varphi = 0$ describes the knife-edge case of endogenous steady state growth if $n = \lambda = 0$.) Substituting the definition of φ from (10), one gets the condition

$$1-\alpha_1-\alpha_2-\beta > \alpha_2/\eta \tag{18}$$

appearing in the text. The right hand side is negative since $\eta < 0$. The condition $1 - \alpha_1 - \alpha_2 - \beta > 0$, mentioned in Section 2.1, is therefore sufficient but not necessary for the existence of a steady state. Its necessity emerges in the special case of a small country with $\eta \to -\infty$, however. It is obvious from these arguments that $1 - \alpha_1 - \beta > 0$ is the relevant condition in case of a closed economy with $\alpha_2 = 0$.

Using the definition of φ in (A1) [taking (12) into account] implies

$$g_X = g_C = g_K = g_A = \gamma_1 n + \gamma_2 \lambda, \tag{15}$$

where γ_1 and γ_2 are defined in the text. From equation (6), $g_M = g_p + g_X$. Substituting $g_M = (\eta + 1)g_p + \lambda$ from (8) implies (16).

B Derivation of Equations (20), (21), and (22)

Dividing equation (9) by $L^{\gamma_1} e^{\gamma_2 \lambda t}$ yields

$$x = \frac{X}{L^{\gamma_1} e^{\gamma_2 \lambda t}} = B \left(\frac{A}{L^{\gamma_1} e^{\gamma_2 \lambda t}} \right)^{\beta\varphi} \left(\frac{K}{L^{\gamma_1} e^{\gamma_2 \lambda t}} \right)^{\alpha_1 \varphi}$$
$$\cdot\, L^{\gamma_1(\beta+\alpha_1)\varphi + (1-\alpha_1-\alpha_2)\varphi - \gamma_1} e^{\gamma_2 \lambda (\beta+\alpha_1)\varphi t - \lambda \alpha_2 \varphi t/\eta - \gamma_2 \lambda t} = B a^{\beta\varphi} k^{\alpha_1 \varphi},$$

because, using the definitions of γ_1, γ_2, and φ, it is tedious but straightforward to show that the exponents of L and e equal zero. This proves (20).

From the definition of k in (19),

$$g_k = g_K - (\gamma_1 n + \gamma_2 \lambda), \quad \text{and} \quad \dot{k} = \frac{\dot{K}}{L^{\gamma_1} e^{\gamma_2 \lambda t}} - (\gamma_1 n + \gamma_2 \lambda) k.$$

Substituting (14), using the definition of x in (19), and finally (20) leads to (21).

From equations (11) and (20),

$$\frac{\dot{A}}{L^{\gamma_1} e^{\gamma_2 \lambda t}} = B a^{\beta\varphi} k^{\alpha_1 \varphi}. \tag{A2}$$

From the definition of a,

$$\frac{\dot{a}}{a} = \frac{\dot{A}}{A} - (\gamma_1 n + \gamma_2 \lambda) \quad \text{and} \quad \dot{a} = \frac{\dot{A}}{L^{\gamma_1} e^{\gamma_2 \lambda t}} - (\gamma_1 n + \gamma_2 \lambda) a.$$

Substituting (A2) yields (22).

C The Isoclines in Figure 1

The exponents $\beta\varphi$ and $\alpha_1\varphi$ in (21) and (22) are positive due to the assumptions about the parameters in (1) and (3), which imply that $\varphi > 0$. Moreover, their sum is smaller than one since

$$\frac{(\beta + \alpha_1)\eta}{\eta - \alpha_2(\eta + 1)} < 1 \quad \iff \quad 1 - \alpha_1 - \alpha_2 - \beta > \alpha_2/\eta,$$

which is (18). From equation (22), the isocline $\dot{a} = 0$ is given by the k-axis and

$$a = \left(\frac{B}{\gamma_1 n + \gamma_2 \lambda} \right)^{1/(1-\beta\varphi)} k^{\alpha_1 \varphi/(1-\beta\varphi)}.$$

Thus, the isocline goes through the origin. Since $(\alpha_1+\beta)\varphi<1$,

$$\frac{\partial a}{\partial k}\Big|_{\dot a=0}=\frac{\alpha_1\varphi}{1-\beta\varphi}\left(\frac{B}{\gamma_1 n+\gamma_2\lambda}\right)^{1/(1-\beta\varphi)}k^{[(\alpha_1+\beta)\varphi-1]/(1-\beta\varphi)}>0,$$

and

$$\frac{\partial^2 a}{\partial k^2}\Big|_{\dot a=0}=\frac{\alpha_1\varphi[(\alpha_1+\beta)\varphi-1]}{(1-\beta\varphi)^2}\left(\frac{B}{\gamma_1 n+\gamma_2\lambda}\right)^{1/(1-\beta\varphi)}k^{[(\alpha_1+2\beta)\varphi-2]/(1-\beta\varphi)}<0.$$

Moreover,

$$\lim_{k\to 0}\frac{\partial a}{\partial k}\Big|_{\dot a=0}=\infty,\quad \text{and}\quad \lim_{k\to\infty}\frac{\partial a}{\partial k}\Big|_{\dot a=0}=0.$$

Thus, the locus $\dot a=0$ has the concave shape shown in Figure 1.

A similar procedure shows that the isocline $\dot k=0$ goes through the origin, is positively sloped, and convex with $\lim_{k\to 0}\frac{\partial a}{\partial k}\big|_{\dot k=0}=0$ and $\lim_{k\to\infty}\frac{\partial a}{\partial k}\big|_{\dot k=0}=\infty$. These results imply that a unique equilibrium $(\bar k,\bar a)$ exists in the positive orthant.

D International Borrowing

In order to show that the principal conclusions of the model do not depend on the assumption of balanced trade, it is sufficient to consider the extreme case in which all imports of the intermediate product are financed by capital imports. That is, the home country does not export goods in exchange for its imports but accumulates external debt. If imports grow at the rate κ such that $M(t)=M_0e^{\kappa t}$, external debt does also grow at this exponential rate. Inserting $M(t)$ into the production function (1), differentiating logarithmically and solving for g_X-n yields

$$g_{C/L}=g_{X/L}=\frac{\beta-\alpha_2}{1-\alpha_1-\beta}n+\frac{\alpha_2}{1-\alpha_1-\beta}\kappa,$$

where the condition $g_X=g_A=g_K$ in a steady state has been used. Thus, supposing that $1-\alpha_1-\beta>0$, if $\beta<\alpha_2$, the per capita growth rate falls in n, although it can still be positive due to capital imports growing at the rate κ.

E R&D-Driven Growth

All non-scale growth models of closed economies display a similar long-run behavior of growth rates, independent of the particular engine of growth. This appendix gives an R&D interpretation of the present model in order to show that the results do not critically depend on learning by doing. Leaving the microeconomic foundations of R&D aside (as in Eicher and Turnovsky, 1999, e.g.), this task is most easily accomplished by the assumption that new ideas are produced using the same technology as the final product. The variable A is now interpreted as the measure of existing ideas, which can be used as a public input in both sectors. It is just the allocation of the private inputs which makes a difference compared to the case of learning by doing. Denoting the inputs used in producing X and new ideas $\dot A$ by the indices X and A, respectively, the production functions are

$$X=A^{\beta}K_X^{\alpha_1}M_X^{\alpha_2}L_X^{1-\alpha_1-\alpha_2},$$
$$\dot A=A^{\beta}K_A^{\alpha_1}M_A^{\alpha_2}L_A^{1-\alpha_1-\alpha_2},$$

where $K_X + K_A = K$ (similarly for the other inputs). In models following Romer (1990), A enters the production function due to the productivity enhancing effect of increasing product variety. Performing Romer's calculations for the present model leads to the additional parameter restriction $\beta = 1 - \alpha_1$. This restriction implies that the condition (18) for non-scale steady state growth reduces to $\eta > -1$.

Given this production technology and the assumption of perfect competition (apart from the intermediate product sector, which is not considered here), the analysis of Section 2 carries over to the present model. In particular, as the shares of the private factors allocated to the two sectors must be constant in a steady state, the long-run growth rates will be identical to those of the learning by doing model. It follows that all theoretical conclusions under the additional restriction $\beta = 1 - \alpha_1$ are valid even in the case of R&D-driven growth. Using $\beta = 1 - \alpha_1$ and $\eta > -1$ in Propositions 2 and 3, however, shows that while $\partial g_{C/L}/\partial n$ may be positive as well as negative, $\partial g_{Y/L}/\partial n$ is always negative. Although the latter result constrains the applicability of this type of R&D model, it should be noted that the condition $\eta > -1$ follows from the special assumption that R&D uses the same technology as the final product sector. In addition to the fact that the parameter values of β and η in Section 2 cannot be used in the R&D-setting, it is also less plausible to assume that an intermediate product constraining the growth rate is not produced at home in the context of industrialized countries engaging in R&D. A more plausible interpretation would be that M is a natural resource which is not available at home (as in Khang, 1968). In summary, while the model presented in Section 2 is concerned with developing countries, similar methods may be applied with respect to the analysis of industrialized countries under the assumption of R&D-driven growth.

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