The Implications of Rybczynski's Theorem for Government Spending, Learning by Doing, and Labor Mobility

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Abstract. The evolution of a small open economy or region with labor mobility and dynamic scale economies in the high-tech sector is analyzed using the neoclassical 2×2-model. Government services are inputs to private production and influence specialization according to Rybczynski's theorem. This effect is reinforced by dynamic scale economies. Empirically observed differences in regional development and specialization are explained by diverging government policies and/or history-dependent factors. Despite of diverging wage rates, diversified regions and regions specialized in low-tech production may coexist with a common level of per capita incomes.

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JEL Classification R11, F11, H72.

1 Introduction

Static and dynamic economies of scale play an important role in the explanation of empirically observed differences in regional development. According to Fujita et al. (1999, p. 2), the dramatic spatial unevenness of the real economy necessarily involves some form of increasing returns. Empirically, there are considerable differences with respect to specialization and technology levels even between regions within a single country. If people are allowed to migrate between regions, as e.g. in the European Union, economic reasoning suggests that the standards of living should equalize despite of such differences in the respective regions. Certainly, several institutional circumstances (such as the German *Länderfinanzausgleich*) are partly responsible for uneven development. It will be shown, however, that even in the absence of transfers, transport costs and other barriers to migration, the coexistence of low-tech and diversified high-tech regions sharing a common equilibrium level of per capita incomes can be explained if dynamic scale economies and possibly different amounts of regional government spending are taken into account.

As far as (dynamic) scale economies in a regional setting (taking migration into account) are concerned, the theoretical analyses are generally restricted to basically linear production technologies explicitly considering just labor as the only input (e.g. Premer and Walz, 1994; Fujita et al., 1999, ch. 5). These approaches neglect relative factor endowments and therefore an important determinant of regional specialization that is emphasized in the theory of international trade and which empirically explains a significant amount of the interregional distribution of production activities (Kim, 1999). An exception is Premer (1994, ch. D), who considers both factor endowments and learning by doing (LBD) as sources of regional comparative advantages, without analyzing the effects of changing factor endowments on the learning opportunities in various regions, however. According to Rybczynski's theorem (1955), changes in factor endowments at constant output prices influence the production structure of an economy or region in a predictable manner depending on the relative factor intensities of the respective industries. If production of an industry is subject to dynamic scale economies, such changes in factor endowments will therefore have long-run effects on acquired comparative advantages through LBD.

The present paper considers a two-sector-two-input model of a small open region that captures these issues.[1] The region's labor force endogenously changes depending on per capita income levels. The regional government provides a composite good of (infrastructure) services as an input to private production. Dynamic scale economies due to learning by doing occur in the sector that uses government services relatively intensively. The specification of government services is taken from Barro's (1990) one-sector-model of endogenous growth.[2] In contrast to Barro (1990), however, the accumulation of physical capital is neglected since emphasis is on labor mobility and LBD. While neglecting the accumulation of capital is a critical assumption in dynamic economic models, it should be noted that the present framework nevertheless goes ahead the usual practice in models of LBD and international or interregional trade which usually consider just one input (e.g. Krugman, 1987; Lucas, 1988; Matsuyama, 1992; Premer and Walz, 1994; Torvik, 2001) or simplify matters otherwise, e.g. by restricting the analysis to a small open economy without labor mobility (Bardhan, 1971; Wong and Yip, 1999).

Among the main results of the model is that government spending influences regional development according to the predictions of the Rybczynski theorem, which are reinforced by the dynamic effects of LBD. If the sector that uses government services relatively intensively (the high-tech sector) exhibits external dynamic scale economies, government spending on infrastructure services can be used to influence the allocation of resources in favor of this sector and therefore regional specialization and development. It is a striking result that, if a diversified equilibrium (i.e., an equilibrium where both goods are produced in positive quantities) is asymptotically stable, a higher value of government spending increases the region's equilibrium population independent of whether government spending is statically too low, efficient, or too high, whereas this result does not hold with respect to the equilibrium value of the knowledge stock (the cumulated experience from LBD). As the region's per capita income in equilibrium coincides with the steady state per capita incomes in other regions by definition, its equilibrium value is independent of government policy. As a result, however, the equi-

[1] Although the notion *region* will be used in the sequel, the model could similarly be interpreted as describing a small open country with a mobile labor force.

[2] Barro's formulation of government expenditure has been generalized in several ways, e.g. by allowing for consumption expenditures, cf. Lau (1995). It is the simplicity of Barro's original formulation which makes it suitable for the present model, however, which is neither concerned with long-run growth nor with an explicit analysis of fiscal competition.

librium wage rates generally differ unless government spending is statically efficient in all regions. Hence, such differences in regional development can be explained as being the result of different policies without having to rely on history-dependent factors. This explanation generalizes the previous literature on diverging development due to LBD, which without paying attention to the Rybczynski theorem has to rely on unstable symmetric equilibria (i.e., equilibria with all regions or countries being identical, especially with respect to their knowledge stocks). E.g., while Premer and Walz (1994) show that the symmetric equilibrium in their model *is* actually unstable, Boldrin and Scheinkman (1988) in a two-country-model *assume* instability. The diversified equilibrium in the present model can be stable as well as unstable, and even cyclical behavior is possible, depending on parameters.

Of course, if all regions were initially identical and at a symmetric, diversified, unstable equilibrium, small accidental historical events would determine the long-run pattern of comparative advantage and diverging regional developments even in the present model. Whereas in case of stable diversified equilibria different amounts of government spending do not have extreme effects on regional development (e.g., a diversified region remains diversified after a slight increase in government spending), regions may experience a radically uneven development in case of unstable or non-existing diversified equilibria. E.g., after a slight increase in government spending an unstable equilibrium will not be reached again. Numerical examples show that a region may loose its knowledge stock completely under such circumstances. It then either also looses its labor force and starves to death or it ends up completely specialized in the production of the good without learning potential. Very high or low levels of government spending can be responsible for the instability or the non-existence of diversified equilibria.

The model and its basic implications are presented in Section 2. In Section 3.1, the local dynamics in the vicinity of a diversified steady state and the comparative statics of such an equilibrium are analyzed. It is also shown that cycles emerge for particular parameter values. The model's global dynamic behavior is considered in Section 3.2 by means of phase diagrams. Some proofs are relegated to appendices. The final section offers concluding remarks.

2 The Model

2.1 Firms

There is much empirical evidence that a high degree of externalities is associated with learning by doing (LBD) (cf. e.g. Lieberman, 1984). As an approximation to reality, it is assumed that learning is purely external on the firm level, which enables a setting of perfect competition. Recent empirical findings of Branstetter (2001), who is mainly concerned with knowledge spillovers from R&D, however, nevertheless suggest that these spillovers are primarily intranational and not international in scope. These findings are adopted to an interregional setting by assuming that LBD is internal to the region.

All profit maximizing firms producing the same commodity share the same production function that is homogeneous of degree one in labor, v, and government services, c, and satisfies the neoclassical properties (i.e., twice continuous differentiability, positive, decreasing marginal productivities and the Inada-conditions). These assumptions imply that both inputs are essential for production and that the aggregated production function of each sector under perfect competition coincides with the respective function at the firm level.[3] The aggregated production functions are

$$x_1 = q^n f_1(v_1, c_1),$$
$$x_2 = f_2(v_2, c_2),$$

where x_i is output of sector i ($i \in \{1,2\}$), v_i and c_i denote the labor and services input in sector i, respectively, and q^n is the constant elasticity of learning function, which has been fitted very well empirically (e.g. Lieberman, 1984). Cumulative production in the high-tech sector 1,

$$q(t) = q(0) + \int_0^t x_1(\tau)\, d\tau,$$

serves as the learning index and may be interpreted as the region's knowledge stock. Therefore, the time derivative is $\dot{q} := dq/dt = x_1$. All variables depend on time, t, but for the sake of notational convenience the time index t will usually be dropped.

Assuming a completely inelastic supply of labor, v, at each point in time and an amount of c supplied by the region's government, which charges a competitive rental rate, the full employment conditions are $v_1 + v_2 = v$ and $c_1 + c_2 = c$. In competitive equilibrium, services intensity in sector i, $k_i := c_i/v_i$, depends on the ratio of the wage rate to the user price of government services, ω. The assumption that sector 1 uses government services relatively intensively can be written as $k_1(\omega) > k_2(\omega)$ for all $\omega > 0$. This assumption implies that the transformation frontier is strictly concave. The sectoral supply functions may be derived by solving the problem of revenue maximization:

$$r = R(pq^n, v, c) := \max_{v_i, c_i \geq 0,\, i=1,2} \{pq^n f_1(v_1, c_1) + f_2(v_2, c_2) \mid v_1 + v_2 \leq v,\, c_1 + c_2 \leq c\}, \quad (1)$$

where $p \in (0, \infty)$ is the relative price of the first commodity in terms of the second commodity and revenue $r = R(pq^n, v, c)$ is measured in units of commodity 2. Since positive marginal productivities have been assumed, the full employment conditions will be met by the solution. It is obvious from this representation of the region's revenue function, $R(pq^n, v, c)$, that the optimum values of v_i and c_i depend on v, c, and pq^n. Substituting into the functions f_i yields the industry supply schedules

$$x_1 = X^1(p, q, v, c) = q^n Z^1(pq^n, v, c), \quad (2)$$
$$x_2 = X^2(p, q, v, c) = Z^2(pq^n, v, c), \quad (3)$$

where $z_i = Z^i(pq^n, v, c)$ are the *activity levels* defined by $x_1 \equiv q^n z_1$ and $x_2 \equiv z_2$. Since the production functions are homogeneous of degree one in v and c, so is the revenue function. As the transformation frontier is strictly concave and the production functions are twice continuously differentiable, the implicit function theorem implies that the revenue function and the supply functions are twice and once continuously differentiable, respectively, in case of interior solutions.

[3]This section mentions some results that are needed for the remainder of the paper without proof or reference as they can be found in any standard text on international trade, e.g. Woodland (1982).

It should be noted that r in this model corresponds to *gross output* (in units of the second commodity), which includes the value of government services. The latter must be subtracted from r to get regional income (value added).

2.2 Government

Following Barro (1990), it is assumed that the government does not produce but just buys a flow of output from sector 1 and transforms this output one-to-one into a kind of composite good including services of highways etc. Empirical evidence suggests that services from government infrastructure are of particular importance for productivity (Aschauer, 1989). In a small region or country with an exogenously given relative price p of good 1 that is freely traded interregionally or internationally, the production of sector 1 does not restrict the government's provision of c. As in Barro (1990), government services are *not* public goods, that is, c is rival in use. Moreover, the government *charges* competitive rental rates. These factor payments are *directly redistributed* to the households. Therefore, these factor payments and transfers to the households are not included in the government's balanced budget constraint, which can be written as $pc = \tau R(pq^n, v, c)$, where τ denotes the endogenous tax rate on gross output used to finance the provision of c. The tax rate is always adjusted to satisfy

$$\tau = \frac{pc}{R(pq^n, v, c)}.$$

It follows that $1 - \tau = (r - pc)/r$. Thus, regional value added is

$$(1 - \tau)R(pq^n, v, c) = R(pq^n, v, c) - pc.$$

The assumption of an endogenous tax rate τ, which simplifies the theoretical analysis considerably, may seem unrealistic at first sight. It should be noted, however, that regional governments often try to stick to their levels of expenditure even if tax revenue declines. Since there is no possibility of deficit spending in the present model, this implies an endogenous tax rate. The implications of the model would be similar if governments left the tax rate unchanged but could adjust public debt instead.

The efficient level of c in a *static sense* is found by maximizing $R(pq^n, v, c) - pc$ with respect to c. Since $R(pq^n, v, c)$ is concave in c due to the concavity of the underlying production functions, the condition

$$R_c(pq^n, v, c) = p \tag{4}$$

is necessary and sufficient for a maximum (partial derivatives are denoted by subscripts). It follows from the Inada conditions that equation (4) has a solution with a positive value of $R(pq^n, v, c) - pc$ which, however, need not be unique. The economic interpretation of condition (4) is as follows. If the dynamic effects of LBD are neglected, government services should be expanded as long as they yield a higher return than their opportunity costs, which equal p.

The possibility of a non-unique solution of equation (4) deserves special attention. Fig. 1 plots the concave, positive revenue function $R(pq^n, v, c)$ against c for given values of pq^n and v. If $c \leq \underline{c}$, the region completely specializes in the production of the labor intensive good 2, and if $c \geq \overline{c}$, it completely specializes in the production of good 1. In these cases, $R_c(pq^n, v, c)$ equals $\partial f_2(v,c)/\partial c$ or $pq^n \partial f_1(v,c)/\partial c$, respectively. Thus, the Inada conditions carry over to the function $R(pq^n, v, c)$ for these intervals. The interval $(\underline{c}, \overline{c})$ corresponds to the cone of diversification (cf. e.g. Woodland, 1982), where both goods are produced in positive quantities. It follows from the factor price equalization theorem that $R_c(pq^n, v, c)$ [as well as $R_v(pq^n, v, c)$] is constant in this interval. Since the respective sectoral factor intensities at the margins of complete specialization are the same as those in the cone of diversification, the function $R(pq^n, v, c)$ is differentiable in c (and v) and the factor prices are unique (for an analytical proof, cf. Chipman, 1987). It should be noted that it is just the constancy of marginal productivities in the cone of diversification that matters here. Due to the possibility of different knowledge stocks, q, factor prices need not equalize between regions even if all the other assumptions of the factor price equalization theorem are met (Bardhan, 1971).

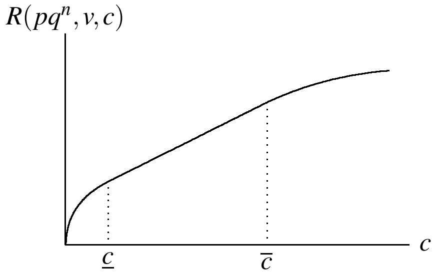

Figure 1. Revenue as a function of services

These properties of the revenue function imply that, for given values of p, q^n and v, the solution of equation (4) will probably result in a value of c that corresponds to complete specialization in the production of one of both goods. The case of a diversified region where condition (4) is met should therefore be considered as a theoretical benchmark case.

2.3 Households

Since the accumulation of physical capital is neglected, rational households just maximize their instantaneous utility functions at every point in time subject to the aggregate budget constraint $px_1^d + x_2^d = R(pq^n, v, c) - pc$, where x_i^d is consumption of good i. (Notice that households receive the factor payments of firms for labor and government services (via transfers) and are taxed by an amount of $\tau r = pc$.) Since a small region with free trade at given prices is considered, there is neither need to derive the demand functions explicitly nor to worry about the aggregation of preferences. Any bundle of goods which maximizes individual utilities subject to the budget constraint is available by interregional trade.

What matters are the welfare effects of public expenditure policy with respect to government services c. Supposing strong monotonicity of preferences, it is well known that aggregate welfare at any point in time as measured by the Samuelson criterion rises with $r - pc$. To account for population changes, however, regional income must be replaced by per capita income.[4] Therefore, the decision to migrate will depend on

[4]Due to the dynamic features of the model, these measures are not as reasonable with respect to the

per capita income $(r-pc)/v = (r/v) - pk$ compared to the per capita income of other regions, where $k := c/v$ denotes the aggregate ratio of government services to labor in the home region.

2.4 Dynamics

The dynamic evolution of the knowledge stock q follows the differential equation

$$\dot{q} = q^n Z^1(pq^n, v, c) - \rho q, \tag{5}$$

where it is recalled that the relative price p of good 1 in terms of good 2 is exogenously given to the small region under free trade. The former specification $\dot{q} = x_1$ of LBD is thus modified by taking the depreciation of knowledge at the rate $\rho \in (0,1)$ into account. This assumption, adopted from Bardhan (1971), is reasonable and enables the analysis of steady states.

The migration of labor depends on the differences in per capita incomes in various regions. Due to migration costs the adjustment to different per capita incomes does not occur infinitely fast but takes time. This is mathematically captured by the following differential equation:

$$\dot{v} = \alpha v[(R(pq^n, v, c)/v) - p(c/v) - y^*] = \alpha[R(pq^n, v, c) - pc - y^* v], \tag{6}$$

where y^* is the given mean of the per capita incomes in other regions at their steady states and $\alpha > 0$ is a parameter influencing the speed of adjustment.[5] The multiplication of the difference of per capita incomes by v captures the idea that, other things being equal, the flow of migrants rises with the size of the region.

Since under free trade prices of goods are equal in all regions and there are no profits, the comparison of the per capita incomes in terms of good 2 as the basis for the decision to migrate is perfectly rational. Using the current per capita income instead of the present value of the future path of per capita income, however, may be interpreted as if households were myopic. This is not the case if it is supposed that households are not aware of the dynamics of the learning index q, which has already been assumed to be an external effect. Under these circumstances, households consider q as a constant; equation (6) implies then that per capita income will never fall short of (exceed) y^* if it exceeds (falls short of) y^* at any point in time. (This does not hold for the complete system comprised of equations (5) and (6).) Thus, equation (6) is consistent with rational behavior and limited information.

Note that the wage rates may differ across regions in equilibrium unless condition (4) is met in every region. In the latter case, the homogeneity of degree one of R in (v,c) implies that $r = R_v v + R_c c = R_v v + pc$ and therefore $R_v = (r-pc)/v = y^*$ in equilibrium. Thus, equal per capita incomes and equal wage rates would be equivalent. Basing the

welfare integrated over time paths. This issue will briefly be discussed in Section 4.

[5] Using the mean of per capita incomes at their steady states implies that the per capita incomes in all other regions coincide. This is certainly a restrictive assumption, which simplifies the analysis considerably, however. If the other regions were not at their steady states and per capita incomes differed, equation (6) would be hard to justify. E.g., $r - pc > y^* v$ would be possible even if some regions had a higher per capita income than the region under consideration. People would then reasonably leave this region rather than entering it.

decision to migrate on per capita incomes instead of wage rates (as in Dendrinos, 1982, e.g.) has the advantage of providing a possible explanation of wage differentials in an interregional equilibrium which do not depend on migration costs but are the result of diverging governmental expenditure policies.

2.5 Basic Comparative Statics Results and Summary of Assumptions

The analysis of the dynamical system in the variables q and v comprising the equations (5) and (6) makes use of the following facts (cf. e.g. Woodland, 1982). The derivative of the supply function (2) with respect to p or q is

$$X^1_p(p,q,v,c) > 0 \text{ and } X^1_q(p,q,v,c) > 0,$$

except for corner solutions. The signs are reversed for the derivatives of the supply function of good 2. In case of diversification of production, the derivatives with respect to the inputs follow from the Rybczynski theorem:

$$X^1_v(p,q,v,c) < 0 \text{ and } X^1_c(p,q,v,c) > 0 \quad \text{if} \quad k_1 > k_2.$$

The derivative of (5) with respect to q is[6]

$$\frac{\partial \dot{q}}{\partial q} = X^1_q(p,q,v,c) - \rho = \frac{x_1}{q} n(1+\varepsilon) - \rho,$$

where $\varepsilon := (\partial x_1/\partial p)(p/x_1)$ is the price elasticity of the supply of commodity 1. If $\dot{q} = 0$, the substitution of $\rho = x_1/q$ yields

$$\left.\frac{\partial \dot{q}}{\partial q}\right|_{\dot{q}=0} = \frac{x_1}{q}[n(1+\varepsilon)-1]. \qquad (7)$$

The derivatives of the revenue function (1) with respect to the inputs yield the respective marginal productivities in terms of good 2 and therefore the wage rate and the rental rate on c, respectively, which are both positive due to positive marginal productivities. According to Hotelling's lemma, $R_p(pq^n,v,c) = x_1$. Since it can be shown that $R_q(pq^n,v,c)$ is equal to $R_p(pq^n,v,c)np/q$, it is positive if $x_1 > 0$. The revenue function is homogeneous of degree one in (v,c).

To summarize, if the high-tech sector 1 uses government services relatively intensively, all of the results derived so far are implied by the standard assumptions about the production functions of the neoclassical 2×2-model of a small open region in a setting of perfect competition, augmented by LBD in sector 1 with a constant learning elasticity and allowing for the depreciation of knowledge. Government services are private goods and financed by a proportional tax on gross output. Rentals are redistributed to households. As to the demand side, no assumptions other than rationality under limited information have been made nor are they necessary. With respect to labor mobility, it

[6]Note that $x_1 = q^n Z^1(pq^n,v,c)$ implies $\partial x_1/\partial p = q^{2n}\partial z_1/\partial(pq^n)$ and therefore

$$X^1_q := \frac{\partial x_1}{\partial q} = nq^{n-1}z_1 + nq^{2n}\frac{\partial z_1}{\partial(pq^n)}\frac{p}{q} = n\frac{x_1}{q} + n\frac{\partial x_1}{\partial p}\frac{p}{q} = \frac{x_1}{q}n\left(1+\frac{\partial x_1}{\partial p}\frac{p}{x_1}\right).$$

has been assumed that all households in a region are equal and that migration depends on the differences in per capita incomes according to equation (6). All of the following results are based on these assumptions, which will therefore not be repeated each time a proposition is stated.

3 Dynamical Analysis

3.1 Local Analysis of a Diversified Steady State

The remainder of the paper is devoted to the analysis of the differential equations (5) and (6). As a first step, it is assumed that an equilibrium with $\dot{q} = \dot{v} = 0$ and diversified production exists. As will be seen in Section 3.2, the existence of such an equilibrium is possible but not generally assured without further assumptions.

Linearizing the equations (5) and (6) around a diversified equilibrium (q_e, v_e) yields

$$\begin{pmatrix} \dot{q} \\ \dot{v} \end{pmatrix} \approx \begin{pmatrix} x_1[n(1+\varepsilon)-1]/q_e & X_v^1(p,q_e,v_e,c) \\ \alpha R_q(pq_e^n,v_e,c) & -\alpha k_e[R_c(pq_e^n,v_e,c)-p] \end{pmatrix} \begin{pmatrix} q-q_e \\ v-v_e \end{pmatrix}, \qquad (8)$$

where all functions are evaluated at (q_e, v_e). The upper-left element of the matrix follows from equation (7). The relation $r/v = R_c k + R_v$, which follows from Euler's theorem on homogeneous functions, and the steady state condition $[R(pq^n,v,c) - pc]/v = y^*$ have been used to calculate the bottom-right element of the matrix (recall that $k := c/v$). If the Routh-Hurwitz conditions are satisfied for the linearized system, the equilibrium of the original system is locally asymptotically stable. These conditions are

$$\frac{x_1}{q_e}[n(1+\varepsilon)-1] - \alpha k_e[R_c(pq_e^n,v_e,c)-p] < 0,$$

$$-\frac{x_1}{q_e}[n(1+\varepsilon)-1]\alpha k_e[R_c(pq_e^n,v_e,c)-p] - \alpha R_q(pq_e^n,v_e,c)X_v^1(p,q_e,v_e,c) > 0,$$

or, if the coefficient matrix of the linearized differential equation is denoted as A, simply $\mathrm{Tr}(A) < 0$ and $|A| > 0$.

There is no way of deciding whether these conditions are satisfied in general. In fact, stability crucially depends on the signs of the two diagonal elements of A. Suppose at first that the static optimum condition (4) is met in equilibrium, such that $R_c = p$ in (8). Then, $\mathrm{Tr}(A)$ is negative if $n(1+\varepsilon) < 1$ or $\varepsilon < 1/n - 1$.[7] Given condition (4), the second Routh-Hurwitz condition is certainly met, because $R_q(pq_e^n, v_e, c) > 0$ and $X_v^1(p, q_e, v_e, c) < 0$ by Rybczynski's theorem. In summary:

Proposition 1 *Suppose that condition (4) is met at a diversified equilibrium. If* $n(1+\varepsilon) < 1$*, this equilibrium is locally asymptotically stable.*

In the general case, the equilibrium may be stable or unstable. If government spending is below the static optimum level in equilibrium, $R_c - p$ is positive in (8). It is

[7] Several empirical estimates suggest that this condition may or may not be satisfied. E.g., the estimates of n from Lieberman (1984) suggest a value near the so-called 80%-curve, that is $n \approx 0.32$. A value of ε smaller than 2.125 would then satisfy the condition. Estimates of ε are available in Kohli (1991), e.g. These rank from very low (0.001) to very high (7.829) values.

straightforward that the stability conditions may be satisfied even if $n(1+\varepsilon) > 1$. If government spending is too high statically and $R_c - p < 0$, the equilibrium will be stable only if $n(1+\varepsilon)$ is sufficiently, but not too much below one. In summary:

Proposition 2 *Compared to the case of a statically efficient level of services, c, the stability conditions for a diversified steady state are less (more) restrictive if c is statically too low (high).*

Thus, excessive values of government spending tend to destabilize the diversified equilibrium.

In a related one-sector-model with labor and capital movements, Dendrinos (1982) found that oscillatory movements would not be feasible without congestion or social friction effects. This result is not valid regarding the present two-sector-model with LBD. The hypotheses of the following proposition, which is proven in Appendix A, rely on the slopes of the isoclines $\dot{q} = 0$ and $\dot{v} = 0$ in (q, v)-space. These slopes are obtained by implicitly differentiating (5) and (6), respectively:

$$\frac{dv}{dq}\bigg|_{\dot{q}=0} = -\frac{x_1[n(1+\varepsilon)-1]}{qX_v^1(p,q,v,c)}, \tag{9}$$

$$\frac{dv}{dq}\bigg|_{\dot{v}=0} = \frac{R_q(pq^n, v, c)}{k[R_c(pq^n, v, c) - p]}, \tag{10}$$

where it should be recalled that $R_q > 0$ and $X_v^1 < 0$.

Proposition 3 *Suppose that there exists a diversified equilibrium where the isoclines $\dot{q} = 0$ and $\dot{v} = 0$ are both positively sloped or both negatively sloped with $|dv/dq|_{\dot{v}=0} > |dv/dq|_{\dot{q}=0}$. Then there exists a range of values of the adjustment parameter α near $\alpha_0 = (x_1[n(1+\varepsilon)-1])/(q_e k_e [R_c - p])$ for which the general solution of system (5), (6) contains a nontrivial closed orbit.*

Fig. 3 (d) below shows that equilibria meeting the hypotheses of Proposition 3 exist.

Return to the case of an asymptotically stable diversified equilibrium. Since $|A| \geq 0$ is a necessary condition for stability, it is assumed that $|A| > 0$, which merely is a regularity condition. The following is proven in Appendix B.

Proposition 4 *If a diversified equilibrium (q_e, v_e) is locally asymptotically stable and the regularity condition $|A| > 0$ holds, a higher value of c increases the region's equilibrium population v_e. The equilibrium learning index q_e rises, is constant or declines depending on whether c is statically too low, efficient or too high.*

The economic reasoning underlying these results is as follows. If $R_c > p$, government spending is too low from a static point of view. A higher value of c diminishes this inefficiency and changes the allocation of resources in favor of sector 1, which uses government services relatively more intensively than labor (the Rybczynski theorem). The increased production of good 1 implies a growing learning index q. Both effects (increasing c and q) lead to a higher regional per capita income. As a result, an immigration of labor takes place, which in turn lowers the production of good 1 and thereby partially offsets the initial increase in q. The per capita income returns to its initial equilibrium level.

If $R_c = p$, a higher value of c again changes the allocation of resources in favor of sector 1. Since the statically efficient level of c is not unique in the cone of diversification, the increased value of c is still efficient in this sense. The increased production of good 1 implies a growth of the knowledge stock q, which in turn leads to a rising per capita income of the region and therefore an inflow of immigrants. The latter lowers the production of good 1 such that the initial increase in q is offset.

If $R_c < p$, c is statically inefficiently high *and* a further increase lowers q_e; nevertheless, v_e increases. This apparently paradoxical outcome may be explained as follows. The initial effect of increasing government spending is a higher value of q due to the enlarged production of good 1, which reduces the static inefficiency because R_c is constant with respect to c but rises with q (note that $R_{cq} = R_{qc} = R_{pc}np/q = npX_c^1/q > 0$). This implies a positive flow of immigrants (not changing the regional marginal productivity of labor) that in turn more than compensates the initial increase of q (since the production of good 1 goes down). It is important in this regard that value added per capita *rises* with v if $R_c < p$:

$$\frac{\partial(r-pc)/v}{\partial v} = \frac{R_v v - r + pc}{v^2} = \frac{\overbrace{R_v v + R_c c - r}^{=0} + pc - R_c c}{v^2} > 0 \quad \text{if} \quad R_c < p.$$

According to Proposition 2, however, the equilibrium is unlikely to be stable for excessive values of c. The dynamic evolution in case of instability will be considered in the next section.

3.2 Global Analysis of the Dynamical System

While a general analytical solution of the model is impossible, it is simple enough to derive vector fields for various cases using computer algebra software. Some typical phase diagrams computed with *Mathematica* will be analyzed in this section.[8]

All of the following figures assume linearly homogeneous Cobb-Douglas production functions where the production elasticity of labor is $1/5$ in sector 1 and $4/5$ in sector 2, respectively. The parameters $n = 1/3$, $p = 1$, $\alpha = 1$, and $\rho = 0.3$ apply to all figures, which differ just with respect to the values of government services, c, and the mean of the steady state per capita incomes, y^*.

Fig. 2 shows the isoclines $\dot{q} = 0$ (the bold grey curve) and $\dot{v} = 0$ (the bold curve), the transition lines between complete specialization in the production of good 1 (good 2, respectively) and diversification, $S(1)$ ($S(2)$, respectively), and the corresponding vector field generated by equations (5) and (6), respectively. Parameters are $c = 48$ and $y^* = 0.53499$. Recall that diversification is a theoretical benchmark case if condition (4) is met, necessitating to choose a particular value of one of the parameters, here y^*, to get a corresponding graphical presentation. The effects of choosing another value of y^* for the shape of the isocline $\dot{v} = 0$ will be explained when discussing Figure 3 (c) below.

[8] *Mathematica* is a registered trademark of Wolfram Research, Inc. A sample *Mathematica* notebook generating Fig. 2 can be downloaded at `www.uni-siegen.de/dept/fb05/vwlii/public/rlbd-ntb.nb`. In order to show that the phase diagrams computed with *Mathematica* are typical indeed and can to some extend be obtained with paper and pencil, the derivation of Fig. 2 is sketched in an Appendix C available for download at `www.uni-siegen.de/dept/fb05/vwlii/public/rlbd-app.pdf`.

To be brief, the expression *specialization* will always mean *complete specialization* in the sequel. Below the curve $S(1)$ (above $S(2)$), the region specializes in the production of good 1 (good 2). Production is diversified between those curves.[9]

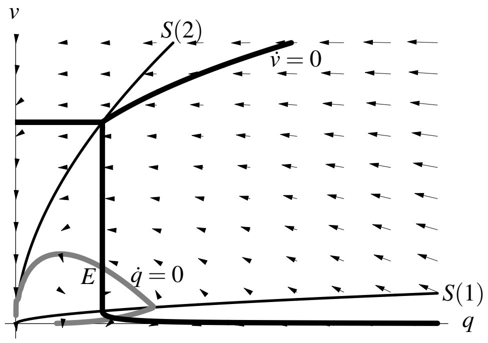

Figure 2. A global vector field (parameters are $c = 48$ and $y^* = 0.53499$)

Apart from $(0,0)$, there are three equilibria, one with diversification, E, one with specialization in the production of good 1, and one with specialization in the production of good 2 (note that $\dot{q} = 0$ on the v-axis). The level of government spending at the diversified equilibrium, E, is statically efficient ($R_c = p$) and hence the slope (10) of $\dot{v} = 0$ is infinite. As the isocline $\dot{q} = 0$ is negatively sloped at E and $X_v^1 < 0$ according to Rybczynski's theorem, it can be seen from (9) that $n(1+\varepsilon) < 1$ must hold. Thus, E is locally asymptotically stable according to Proposition 1. The specialized equilibrium below $S(1)$ can be shown to be a saddle and thus to be unstable – as it is (poorly) indicated by the vector field. The specialized equilibrium where $\dot{v} = 0$ hits the v-axis is also unstable, but it is one-sided stable along the v-axis and may be approached asymptotically if v is initially sufficiently large. Similarly, the origin is not stable but may be approached by some trajectories depending on the initial values. Thus, as both of the specialized equilibria are unstable, the region will probably either reach the diversified equilibrium or it ends up depopulated at the origin.

Figures 3 (a) and 3 (b) differ from Fig. 2 just with respect to the underlying amounts of government spending and thus the available amounts of c, which are both efficient, however. In Fig. 3 (a), $c = 80$ is greater than in Fig. 2, while in Fig. 3 (b), it is smaller ($c = 30$). Even though the amount of c is statically efficient in all three figures along the vertical part of $\dot{v} = 0$, the fate of the region in question will, in the long run, differ completely depending on the level of government spending. While a diversified equilibrium with positive population is likely in case of Fig. 2 (in the sense that the basin of attraction of E is relatively large), the probability of starving to death is much greater in both of the other cases (spiraling in Fig. 3 (a), where the diversified equilibrium is unstable, and monotone in Fig. 3 (b), where no diversified equilibrium exists). The equilibria with specialization in the production of good 2 are one-sided stable along the v-axis and will only be approached for relatively high initial values of v. It depends on

[9] It should be noted that *Mathematica* did not capture the horizontal linear parts of $\dot{v} = 0$ in Figures 2, 3 (a), and 3 (b). Therefore, these lines have been added manually to the figures. *Mathematica* is able to find the corresponding roots of $\dot{v} = 0$ for the respective values of q with its function *FindRoot*, however. The existence of the linear parts in question follows from the fact that $\dot{v} = 0$ has a point in common with $S(2)$ in the respective figures and q does not affect $\dot{v}$ if the region specializes in the production of good 2.

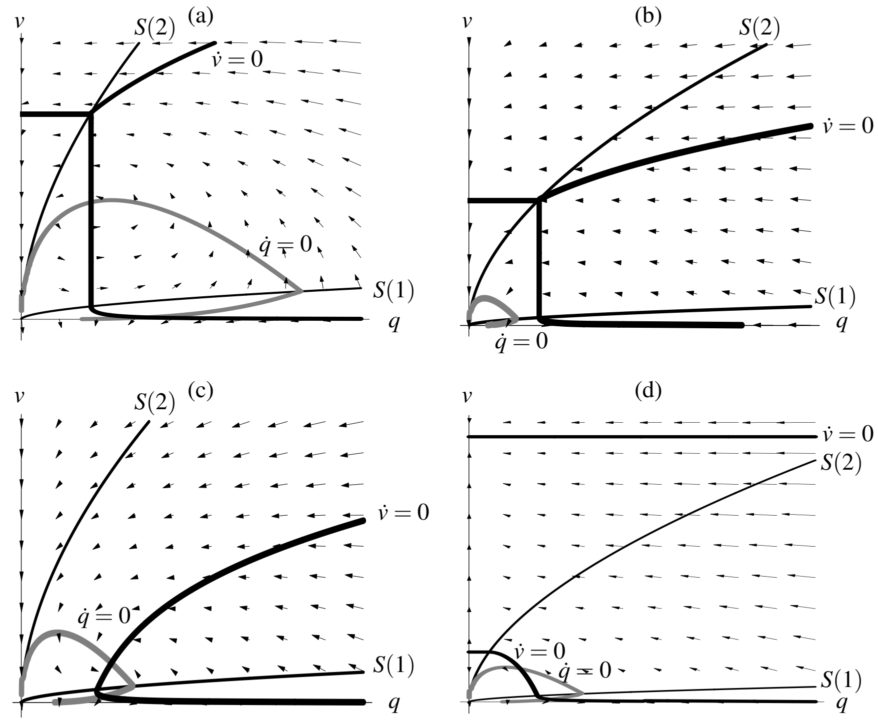

Figure 3. Various global vector fields

the initial values and therefore is in a sense a matter of historical accidents whether in the long run the region specializes in the production of good 2 or becomes depopulated.

Figures 3 (c) and 3 (d) differ from Fig. 2 just with respect to the mean of the per capita incomes in other regions. In 3 (c), $y^* = 0.6$, and in 3 (d), $y^* = 0.5$. A higher value of the mean of the per capita incomes implies that for any given population v, a greater value of q than before is required to return to $\dot{v} = 0$. Since $R_{cq} > 0$ by the Stolper-Samuelson theorem, this implies that the value of R_c on $\dot{v} = 0$ rises with the mean of the per capita incomes. Starting from a situation with $R_c = p$ on $\dot{v} = 0$, a higher value of y^* therefore implies that $R_c > p$ at the new diversified equilibrium. Thus, the equilibrium level of government spending in Fig. 3 (c) is statically too low. Since $R_c > p$ and in accordance with Proposition 2, it is straightforward to determine from the slopes of the isoclines that the diversified equilibrium is locally asymptotically stable. As the specialized equilibrium below $S(1)$ is again a saddle, the conclusions about the probable dynamical development of the region are very similar to those drawn with respect to Fig. 2.

Finally, in Fig. 3 (d), the lower value of the mean of the per capita incomes implies that the level of government spending at the diversified equilibrium is too high from a static point of view. Since $R_c < p$ in this case, the isocline $\dot{v} = 0$ has a negative slope now. As $\dot{q} = 0$ is negatively sloped, it is impossible to decide whether the diversified equilibrium is stable or not relying solely on qualitative information. In the example, the isocline $\dot{v} = 0$ is steeper than $\dot{q} = 0$ at the equilibrium, which for the present case implies that the second Routh-Hurwitz condition, $|A| > 0$, is met. Whether $\mathrm{Tr}(A) < 0$ holds also cannot be seen by means of the figure. However, it is easily shown with *Mathematica* that $\mathrm{Tr}(A) < 0$ for the present parameter values. Thus, the diversified equilibrium in Fig. 3 (d) is locally asymptotically stable. If α was chosen as α_0 in Proposition 3, however, a Hopf bifurcation giving rise to a closed orbit would occur at α_0.

Fig. 3 (d) has been heavily distorted (vertically shrunk) in comparison with the other computed figures in order to capture another interesting characteristic of this example (without reserving too much space). Apart from the origin, the stable diversified equilibrium, and the saddle below $S(1)$, there is another *stable* equilibrium on the v-axis, where the upper part of $\dot{v} = 0$ hits this axis. The basin of attraction of the equilibrium on the v-axis with specialization in the production of good 2 is overwhelming large. In addition, the equilibrium value of v is extraordinarily high in comparison with other equilibria in this and other figures. It follows that some regions could produce no high-tech goods in equilibrium, but nevertheless have a relatively large population that shares the mean of the per capita incomes of other regions producing both goods in the stable diversified equilibrium of Fig. 3 (d).

In summary, the analysis shows that even if the amounts of government spending are equal in several regions and hence the same vector field applies to them, the patterns of specialization and the knowledge stocks may differ considerably between these regions. If, moreover, government spending differs from region to region, even the vector fields describing the dynamic evolution themselves do not coincide anymore. Nevertheless, all populated regions with a great diversity of government spending, knowledge stocks, and population share an equal per capita income in the long run. Since all equilibria with specialization in the production of good 1 are unstable, it is likely to observe regions specialized in low-tech goods and regions with diversified production, but no regions completely specialized in the production of high-tech goods.

4 Concluding Remarks

Using the 2×2-model is an important generalization of approaches to regional development restricting attention to either just one input or just one output. Due to the Rybczynski theorem, government spending on services now influences the regional production structure and therefore LBD and migration. As long as a diversified equilibrium is asymptotically stable, a higher value of government spending increases the region's equilibrium population. Excessive values of government spending, however, tend to destabilize the diversified equilibrium. In case of an unstable or non-existent equilibrium, cyclical behavior of population and the knowledge stock is possible, and complete specialization in the production of the good without learning potential becomes likely. It is also important to recognize the implications of the constancy of factor prices in the cone of diversification. E.g., Figures 2, 3 (a), and 3 (b) differ just with respect to government spending, which is statically efficient in all three cases. Without LBD, there would be no incentives for migration in such a case, since free trade in goods equalized factor prices and per capita incomes. In the presence of LBD, however, the factor price equalization theorem ceases to hold.

Further research should include the explicit introduction of many small regions to allow for the determination of a full general equilibrium with respect to prices and per capita incomes. An interesting question arises from the distinction of centralized versus decentralized budgets of the regional governments. Moreover, it should be possible to consider the model with government services as a public good. Other issues include different kinds of technological change, as discussed in the theory of endogenous economic growth, and the accumulation of capital. The latter is of particular importance, since different rates of accumulation influence the production structure of the respective regions according to the Rybczynski theorem and therefore are important for LBD. Some of these possible extensions would require computable simulation models, however.

Another extension is the formulation and solution of the dynamic optimization problem as faced by the government. In fact, the government should consider the dynamic paths of the knowledge stock and population and should try to maximize welfare integrated over time. The primary concern here have been the positive features of the model, however, and it is the author's conviction that models involving dynamic optimization by governments are only of a limited value to *positive* economics. (Nevertheless, these models are of great importance in *normative* economics.) It should be noted that due to the model's structure with external LBD and no assets that a household could accumulate there is no need to consider a dynamic optimization approach of individual households.

Appendix

A Proof of Proposition 3

The emergence of a nontrivial closed orbit can be proven by the *Hopf bifurcation theorem*. A Hopf bifurcation occurs if the following conditions are satisfied: first, a dynamical system depending on a parameter α has an equilibrium at α_0, where, second, the Jacobian with respect to the variables evaluated at the equilibrium has a simple pair of purely imaginary conjugated eigenvalues and no other eigenvalues with zero real part, and, third, these eigenvalues as a function of α cross the imaginary axis with non-zero speed at $\alpha = \alpha_0$. Given these hypotheses, there exists a range of values of α near α_0 such that a nontrivial closed orbit emerges for these parameter values.[10]

To begin with, observe that the condition that the isoclines $\dot{v} = 0$ and $\dot{q} = 0$ are both positively or both negatively sloped is equivalent to

$$\operatorname{Sgn}[n(1+\varepsilon)-1] = \operatorname{Sgn}[R_c - p] \neq 0, \tag{A1}$$

cf. (9) and (10). Now, if an equilibrium (q_e, v_e) exists where (A1) holds, there is an $\alpha_0 > 0$ defined by

$$\alpha_0 = \frac{x_1[n(1+\varepsilon)-1]}{q_e k_e (R_c - p)}, \tag{A2}$$

where all functions are evaluated at (q_e, v_e). Choosing $\alpha = \alpha_0$ implies $\operatorname{Tr}(A) = 0$ [cf. (8)]. As the eigenvalues of the Jacobian A of system (5), (6) at a diversified equilibrium are $\lambda_{1,2} =$

[10] The stability properties of limit cycles may be analyzed by a condition involving the computation of third order direct and mixed partial derivatives of the involved differential equations (cf. Guckenheimer and Holmes, 1983, p. 152). Since X_q^1 depends on the second order derivatives of the production functions, this would require assumptions on their fourth order derivatives, which have no clear economic interpretation. Therefore, the stability of the closed orbits will not be analyzed.

$0.5\left[\mathrm{Tr}(A) \pm \sqrt{\mathrm{Tr}(A)^2 - 4|A|}\right]$, $\mathrm{Tr}(A) = 0$ implies that there is a pair of purely imaginary conjugated eigenvalues if $|A| > 0$. Now observe that, given that (A1) holds, the condition $|A| > 0$ is equivalent to $\dot{v} = 0$ being steeper than $\dot{q} = 0$ [cf. (9) and (10)]. Hence, under the assumptions of Proposition 3, the first and the second condition of the Hopf bifurcation theorem are satisfied at α_0. Since $x_1[n(1+\varepsilon)-1]/q_e$ and $k_e(R_c - p)$ are independent of α (even indirect, as the equilibrium (q_e, v_e) is independent of α), the derivative of the real part $0.5\mathrm{Tr}(A)$ of the eigenvalues with respect to α at α_0 is simply $-0.5k_e[R_c - p]$, which does not vanish since $R_c - p \neq 0$. Thus, the third condition for a Hopf bifurcation is also satisfied.

B Proof of Proposition 4

Setting equations (5) and (6) equal to zero, differentiating with respect to the endogenous variables q and v and the exogenous variable c, and solving for $\partial q_e/\partial c$ and $\partial v_e/\partial c$ yields:

$$\frac{\partial q_e}{\partial c} = \frac{\alpha(R_c - p)(X_c^1 k_e + X_v^1)}{|A|},$$

$$\frac{\partial v_e}{\partial c} = \frac{\alpha[X_c^1 R_q - x_1[n(1+\varepsilon)-1](R_c - p)/q_e]}{|A|}.$$

As $X^1(p,q,v,c)$ is linearly homogeneous in (v,c), Euler's theorem on homogeneous functions implies $X_c^1 k_e + X_v^1 = \frac{1}{v_e}(X_c^1 c_e + X_v^1 v_e) = x_1/v_e > 0$ at a diversified equilibrium. Together with $|A| > 0$, it follows that $\partial q_e/\partial c \gtreqless 0$ if $R_c \gtreqless p$. The condition $\partial v_e/\partial c > 0$ follows from

$$\begin{aligned}
|A|\frac{\partial v_e}{\partial c} &= \frac{1}{k_e}[\alpha k_e X_c^1 R_q - \alpha k_e \frac{x_1}{q_e}[n(1+\varepsilon)-1](R_c - p)] \\
&= \frac{1}{k_e}[-\alpha k_e \frac{x_1}{q_e}[n(1+\varepsilon)-1](R_c - p) - \alpha X_v^1 R_q + \alpha X_v^1 R_q + \alpha k_e X_c^1 R_q] \\
&= \frac{1}{k_e}|A| + \frac{\alpha R_q}{k_e}(X_c^1 k_e + X_v^1) = \frac{1}{k_e}\left(|A| + \alpha R_q \frac{x_1}{v_e}\right) > 0.
\end{aligned}$$

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References

Aschauer, D. A. (1989): Is Public Expenditure Productive?, *Journal of Monetary Economics*, 23, 177–200.

Bardhan, P. K. (1971): On Optimum Subsidy to a Learning Industry: An Aspect of the Theory of Infant Industry Protection, *International Economic Review*, 12, 54–70.

Barro, R. J. (1990): Government Spending in a Simple Model of Endogenous Growth, *Journal of Political Economy*, 98, S103–S125.

Boldrin, M. and Scheinkman, J. A. (1988): Learning-By-Doing, International Trade and Growth: A Note, in: P. W. Anderson, K. J. Arrow, and D. Pines (eds.), *The Economy as an Evolving Complex System*, 285–300, Reading, MA: Addison-Wesley.

Branstetter, L. G. (2001): Are Knowledge Spillovers International or Intranational in Scope? Microeconometric Evidence from the U.S. and Japan, *Journal of International Economics*, 53, 53–79.

Chipman, J. S. (1987): International Trade, in: J. Eatwell, M. Milgate, and P. Newman (eds.), *The New Palgrave, A Dictionary of Economics*, 922–955, London: Macmillan.

Dendrinos, D. S. (1982): On the Dynamic Stability of Interurban/Regional Labor and Capital Movements, *Journal of Regional Science*, 22, 529–540.

Fujita, M., Krugman, P., and Venables, A. J. (1999): *The Spatial Economy - Cities, Regions, and International Trade*, Cambridge: MIT Press.

Guckenheimer, J. and Holmes, P. (1983): *Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields*, New York: Springer.

Kim, S. (1999): Regions, Resources, and Economic Geography: Sources of U.S. Regional Comparative Advantage, 1880–1987, *Regional Science and Urban Economics*, 29, 1–32.

Kohli, U. (1991): *Technology, Duality, and Foreign Trade: The GNP Function Approach to Modeling Imports and Exports*, New York: Harvester Wheatsheaf.

Krugman, P. (1987): The Narrow Moving Band, the Dutch Disease, and the Competitive Consequences of Mrs. Thatcher: Notes on Trade in the Presence of Dynamic Scale Economies, *Journal of Development Economics*, 27, 41–55.

Lau, S.-H. P. (1995): Welfare-Maximizing vs. Growth-Maximizing Shares of Government Investment and Consumption, *Economics Letters*, 47, 351–359.

Lieberman, M. B. (1984): The Learning Curve and Pricing in the Chemical Processing Industries, *RAND Journal of Economics*, 15, 213–228.

Lucas, Jr., R. E. (1988): On the Mechanics of Economic Development, *Journal of Monetary Economics*, 22, 3–42.

Matsuyama, K. (1992): Agricultural Productivity, Comparative Advantage, and Economic Growth, *Journal of Economic Theory*, 58, 317–334.

Premer, M. (1994): *Integration, interregionaler Handel und wirtschaftliche Entwicklung von Regionen*, Tübingen: Francke.

Premer, M. and Walz, U. (1994): Divergent Regional Development, Factor Mobility, and Non-Traded Goods, *Regional Science and Urban Economics*, 24, 707–722.

Rybczynski, T. M. (1955): Factor Endowments and Relative Commodity Prices, *Economica*, 22, 336–341.

Torvik, R. (2001): Learning by Doing and the Dutch Disease, *European Economic Review*, 45, 285–306.

Wong, K.-Y. and Yip, C. K. (1999): Industrialization, Economic Growth, and International Trade, in: E. K. Choi and B. S. Jensen (eds.), *Economic Growth and International Trade*, 164–182, Oxford: Blackwell.

Woodland, A. D. (1982): *International Trade and Ressource Allocation*, Amsterdam: North-Holland.